**Optimization of utility**

it’s constraint by income.

Derivative of factor that determining the consumption is relative price

**Optimization of production**

It’s constraint by the total labor and capital.

When given level of one production, the remaining one can be maximized.

Derivative of factor determining the labor and capital is the ratio of Lagrangian factor.

The total derivative of the level sets equals 0

Production possibility frontier (every possible combination of x and y should be inside the frontier curve) and efficient production set (straight line ) can be derived mathematically for a given level of x.

**Combination of production and consumption**

One graph shows production possibility frontier and utility level sets and the intersection suggests the possible production and the demand of society. When they barely touch, the slope manifests the ratio of price, which is the insight of “invisible hand” by Adam Smith